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II. Solution by **B. F. FINKEL, A. M., M. Sc., 204 St. Marks Square, Philadelphia, Pa.**

The equation may be written in the form $y=\pm\frac{x}{a}\surd(c^2-z^2)$. If $z=0$, $y=\pm\frac{c}{a}x$, the equation of two lines in the xy-plane passing through the origin; if $y=0$, $z=\pm c$, the equation of two lines in the xz-plane parallel to the x-axis. It is thus seen that the given surface intersects the xz-plane in two parallel straight lines, and the xy-plane in two straight lines passing through the origin. If $x=a$, $y=c^2-z^2$, the equation of a circle. The intersection of the surface with a plane parallel with the yz-plane at a distance $x=k<a$, is an ellipse. If we pass a plane parallel to the xy-plane at a distance $z<c$, the intersection of this plane with the surface and the plane $x=a$, is a triangle whose altitude is a, and base $2y=2a\surd(c^2-z^2)$. Hence we have for the required volume,

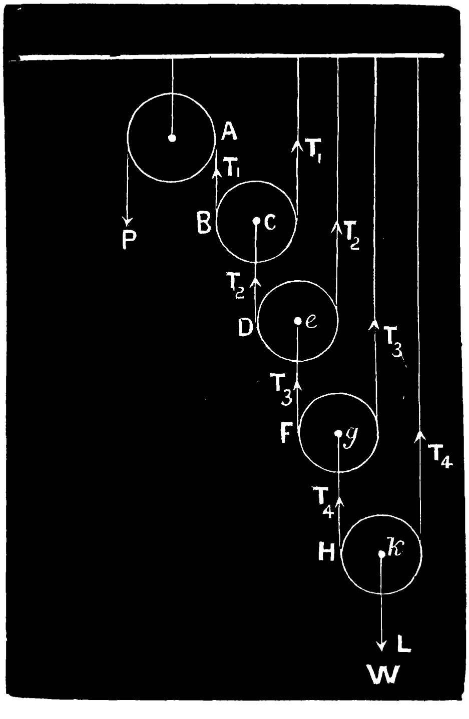
$$V=2\int_0^c 2a\surd(c^2-z^2)dz=4a\left[\frac{z}{2}\surd(c^2-z^2)+\frac{c^2}{2}\sin^{-1}\frac{z}{c}\right]_0^c=\tfrac{1}{2}\pi ac^2.$$

Also solved by W. W. Landis, Dickinson College, Carlisle, Pa.; E. L. Sherwood, Shady Side Academy, Pittsburg, Pa.; William Hoover, Athens, O.; G. B. M. Zerr, Parsons, W. Va.; L. C. Walker, Golden, Col.; J. Scheffer, Hagerstown, Md.; and the Proposer.

MECHANICS.

164. Proposed by **W. J. GREENSTREET. A. M., Editor of The Mathematical Gazette, Stroud, England.**

P balances W on a system of n movable pulleys of equal weight, each hanging by a separate string. If P is moved find the maximum acceleration of W.



Solution by **G B M ZERR, A M, Ph D, Parsons, W Va**

Let T_1, T_2, T_2, ..., T_n be the tensions on AB, CD, EF, ..., GH; x_1, x_2, x_3, ..., x_n the lengths AB, CD, EF, ..., GH. Then dx_1/dt, dx_2/dt, dx_3/dt, ..., dx_n/dt represent the velocities.

Let Q be the additional weight which moves the system, m the weight of each pulley.

Now $P=T_2$, $2T_1=T_2+m$, $2T_2=T_3+m$, ..., $2T_n=m+W$.

$\therefore 2P=2T_1=T_2+m$, $4P=2T_2+2m=T_3+3m$, $2^3P=2T_3+6m=T_4+(2^3-1)m$, ..., $2^{n-1}P=T_n+(2^{n-1}-1)m$, $2^nP=(2^n-1)m+W$.

The equations of motion are

$$\left(\frac{P+Q}{g}\right)\cdot\frac{d^2x_1}{dt^2}=P+Q-T_1,$$

$$\frac{m}{g}\cdot\frac{d^2x_1}{dt^2}=2T_1-m-T_2,$$

$$\frac{m}{g}\cdot\frac{d^2x_2}{dt^2}=2T_2-m-T_3,\quad \frac{m}{g}\cdot\frac{d^2x_3}{dt^2}=2T_3-m-T_4,$$

....

$$\frac{m}{g}\cdot\frac{d^2x_{n-1}}{dt^2}=2T_{n-1}-m-T_n,\quad \left(\frac{m+W}{g}\right)\cdot\frac{d^2x_n}{dt^2}=2T_n-m-W.$$

Eliminating T_1 we get

$$\left(\frac{2(P+Q)}{g}+\frac{m}{2g}\right)\frac{d^2x_1}{dt^2}=2(P+Q)-m-T_2.$$

Now $dx_1/dt=2dx_2/dt=2^2dx_3/dt=.....=2^{n-1}dx_n/dt$.

$\therefore d^2x_1/dt^2=2d^2x_2/dt^2=2^2d^2x_3/dt^2=.....=2^{n-1}d^2x_n/dt^2$.

Eliminating T_2 we get

$$\left(\frac{2^2(P+Q)}{g}+\frac{m}{g}+\frac{m}{2^2g}\right)\cdot\frac{d^2x_1}{dt^2}=2^2(P+Q)-2m-m-T_3.$$

Eliminating T_3 we get

$$\left(\frac{2^3(P+Q)}{g}+\frac{2m}{g}+\frac{m}{2g}+\frac{m}{2^3g}\right)\cdot\frac{d^2x_1}{dt^2}=2^3(P+Q)-2^2-2m-m-T_4.$$

Hence eliminating all the tensions we get

$$\frac{2^n(P+P)}{g}+\frac{m}{g}\left(2^{n-2}+2^{n-4}+2^{n-6}+.....+\frac{1}{2^{n-4}}+\frac{1}{2^{n-2}}+\frac{1}{2^n}\right)\cdot\frac{d^2x_1}{dt^2}$$

$$=2^n(P+Q)-m(2^{n-1}+2^{n-2}+2^{n-3}+.....+2^2+2+1)-W$$
$$=2^n(P+Q)-m(2^n-1)-W.$$

$$\therefore \frac{1}{2^ng}\cdot\frac{d^2x_1}{dt^2}\left(2^{2n}(P+Q)+m(2^{2n-2}+2^{2n-4}+2^{2n-6}+.....+2^4+2^2+1)\right)$$

$$=2^{2n}(P+Q)-m(2^n-1)-W.$$

$$\therefore \frac{1}{2^n}\cdot\frac{d^2x_1}{dt^2}=\frac{[2^n(P+Q)-m(2^n-1)-W]g}{2^{2n}(P+Q)+\frac{1}{3}m(2^{2n}-1)}=\text{acceleration of } W=A.$$

Substituting the value of P we get

$$A=\frac{3.2^nQg}{3.2^{2n}Q+3.2^nW+3.2^n(2^n-1)m+(2^{2n}-1)m}$$

$$=\frac{3.2^nQg}{3.2^{2n}Q+3.2^nW+(4.2^{2n}-3.2^n-1)m}.$$

When Q is infinite A is a maximum$=g/2^n$.